Real Numbers

1) Show that of the numbers n, n+2 and n+4, only one of them is divisible by 3.

2012/2013/2014/2015/2016 [2 marks]

Let n be any positive integer. Then,

$n=3q$ or $3q+1$ or $3q+2$.

If $n=3q$, then $n=3q$ is divisible by 3, $n+2=3q+2$ is not divisible by 3 and also

$n+4=3q+4=3(q+1)$ is not divisible by 3.

If $n=3q+1$, then $n=3q+1$ not divisible by 3, $n+2=3q+1=3(q+1)$ is divisible by 3 and $n+2=3q+1+4=3(q+1)+2$ is not divisible by 3.

If $n=3q+2$, then $n=3q+2$ is not divisible by 3, $n+2=3q+2+2=3(q+1)+1$ is not divisible by 3 and $n+4=3q+2+4=3(q+2)$ is divisible by 3.

The only one, out of n, n+2 and n+4, is divisible by 3.

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2) Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m+1 for some integer m.

2010/2011/2012/2013/2014/2016 [3 marks]

Let α be a positive integer. Then, it can be expressed as 3q or 3q+1 or 3q+2.

Now, we have to show that the square of each of these can be written in the form 3m or 3m+1.

If $\alpha=3q$, we have: $\alpha^2=(3q)^2=3\times3q^2$

$=3m$, where $m=3q^2$.

If $\alpha=3q+1$, then $\alpha^2=(3q+1)^2=9q^{2+}+6q+1=3(3q^2+2q)+1$

$=3m+1$. Where $m=3q^2+2q$.

If $\alpha=3q+2$, then $\alpha^2=(3q+2)^2$

$=9q^2+12q+4$

$=3(3q^2+4q+1)+1$

$=3m+1$, where $m=3q^2+4q+1$.

Thus, α^2 is either of the form 3m or 3m+1.

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3) Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m+1 or 9m+8.

2010/2011/2012/2014/2015/2016 [3 marks]

Let α be any positive integer. Then, it is of the form 3p, 3p+1 or 3p+2. Now we have to show that the cube of each of these can be expressed in the form 9m, 9m+1 or 9m+8.

If $\alpha = 3p$, then $\alpha^3=(3p)^3=9(3p^3) = 9m$, where $m=3p^3$.

If $\alpha = 3p+1$, then $\alpha^3 =(3p+1)^3 = (3p)^3+3(3p)2.1+3(3p).1^2+1^3$

$=27p^3 + 27p^2+9p+1.$

$=9(3p^3 +3p^2+p)+1$

$=9m+1$, where $m=3p^3+3p^2+p$.

If $\alpha=3p+2$, then $\alpha^3 =(3p+2)^3$

$=(3p)^3 + 3(3p)^2 .2+3(3p).2^2+2^3$

$=27p^3 + 54p^2+36p+8.$

$=9(3p^3 +6p^2+4p)+8$

$=9m+8$, where $m=3p^3+6p^2+4p$.

Thus, α^3 is either of the form $9m$, $9m+1$ or $9m+8$ for some integer m.

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4) The traffic lights at three different road crossings change after every 48 seconds 72 seconds and 108 seconds respectively. If they all change simultaneously at 8:00 hours, then at what time will they again change simultaneously?

2014/2015/2016 [3 marks]

$48=2\times2\times2\times2\times3$

$72=2\times2\times2\times3\times3$

$108=2\times2\times3\times3\times3$

So, LCM (48,72,108) = $2\times2\times2\times2\times3\times3\times3=432$.

Now, 432 seconds = 7 minutes and 12 seconds.

Hence, they will again simultaneously change at 8:07:12 hours.

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5) Prove that $\sqrt{3}$ is an irrational number.

2013/2014/2015/2016 [3 marks]

If possible , let $\sqrt{3}$ be rational number.

Let $\sqrt{3} = \frac{a}{b}$, where a and b are co-prime and $b\neq0$.

So, $3=a^2/b^2$ or $3b^2 = a^2$. ..... (1)

Also a^2 is divisible by 3 $\rightarrow$ a is divisible by 3. ......(2)

Let $a=3c$, where c is an integer.

$\rightarrow$ $a^2=9c^2$

$\rightarrow$ $3b^2=9c^2$ [substituting $a = 3c$ in (1)]

Or $b^2=3c^2$.

$\rightarrow$ b^2 is divisible by 3 and hence b is divisible by 3. ....(3)

From (2) and (3), we conclude that 3 is common factor of and b.

But, we assumed that a and b are co-prime. Hence, a contradiction.
$\therefore$ $\sqrt{3}$ is not a rational number, i.e., it is an irrational number.

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6) Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal.

2014/2015/2016 [3 marks]

We have : $\frac{1}{7}$

For terminating decimal expression , 7 should be removed from the denominator.

Further, for decimal expansion to terminate after 2 places of decimal, there should be $2^2.5^2$ in the denominator.

So, smallest positive rational number to obtain a decimal expansion terminating after 2 decimal places is $\frac{7}{2^2 \cdot 5^2} = \frac{7}{100}$.

[Note that by multiplying $\frac{1}{7}$ *by* $\frac{7}{2^2}$ *or* $\frac{7}{5^2}$ will also give a decimal expansion terminating after 2 decimal places. But smallest positive rational number is $\frac{7}{100}$]